MATH 2850: HIGHER ORDER LINER ODEs AND UNDETERMINED COEFFICIENTS (REVISITED)

RECALL: Linear IVPs:

- First Order: $y' + p(x)y = f(x)$, $y(x_0) = y_0$

  We were guaranteed unique solution if p and f are continuous on an open interval containing x_0.

- Second Order: $y'' + p(x)y' + q(x)y = f(x)$, $y(x_0) = y_0$, $y'(x_0) = y_1$

  We were guaranteed unique solution if p, q and f are continuous on an open interval containing x_0.

IN GENERAL: An n^{th} order linear IVP of the form

$$y^{(n)} + p_1(x)y^{(n-1)} + p_2(x)y^{(n-2)} + \ldots + p_{n-1}(x)y' + p_n(x)y = f(x)$$

subject to:

$$y(x_0) = y_0, y'(x_0) = y_1, \ldots, y^{(n-2)}(x_0) = y_{n-2}, y^{(n-1)}(x_0) = y_{n-1}$$

has a unique solution provided p_1, p_2, $\ldots p_n$ are continuous in an open interval containing x_0.

As usual, the solution to this IVP $y = y_c + y_p$ where y_c is the solution to the associated homogeneous equation:

$$y^{(n)} + p_1(x)y^{(n-1)} + p_2(x)y^{(n-2)} + \ldots + p_{n-1}(x)y' + p_n(x)y = 0$$

QUESTION: How many arbitrary constants do you expect in the solution to

$$y^{(n)} + p_1(x)y^{(n-1)} + p_2(x)y^{(n-2)} + \ldots + p_{n-1}(x)y' + p_n(x)y = 0$$

NOTE: All of the theory from first and second order linear equations extend!

EXAMPLE: Find the general solution to $y''' - 3y'' + 4y = 0$.

Ans: $y = c_1 e^{-x} + c_2 e^{2x} + c_3 x e^{2x}$

EXAMPLE: What would the **form** of the particular solution to $y''' - 3y'' + 4y = \sin(x) + 7\cos(x) - 9e^{-x}$ take?

Ans: $y = A\sin(x) + B\cos(x) + Cxe^{-x}$.

EXAMPLE: Solve $y''' - 3y'' + 4y = \sin(x) + 7\cos(x) - 9e^{-x}$ using the method of undetermined coefficients.

Ans: $y = c_1 e^{-x} + c_2 e^{2x} + c_3 x e^{2x} + \cos(x) - xe^{-x}$

EXAMPLE: Solve the IVP: $y''' - 3y'' + 4y = \sin(x) + 7\cos(x) - 9e^{-x}$, $y(0) = 9$, $y'(0) = -2$, $y''(0) = 10$

Ans: $y = 5e^{-x} + 3e^{2x} - 2xe^{2x} + \cos(x) - xe^{-x}$

EXAMPLE: Suppose the auxiliary equation for a homogeneous constant coefficient ODE is:

$$m^2(m-1)^3(m^2-2m+5)^2$$

What is the order of the DE?

What is the form of the general solution?

Ans: $y = c_1 + c_2x + c_3e^x + c_4xe^x + c_5x^2e^x + e^x\left[c_6\sin(2x) + c_7\cos(2x)\right] + xe^x\left[c_8\sin(2x) + c_9\cos(2x)\right]$

HOMEWORK: Pg. 483: 1-25 e.o.o., Pg. 495: 1 - 67 every third odd